Kenneth R. Smith
Bayview Strategies, LLC
PMB 176
6068 Hwy 98 W, Suite 1
Hattiesburg, MS 39402

Bayview Strategies, LLC

Scott Gallagher
Ubetcha Games
Carver, MA 02330

Dear Sir:

Pursuant to your request for a mathematical review of the proposed casino game "High Survivor", I have prepared the following brief summary.

The game can operate with three slightly different rule-sets, depending on how many players are present.

Case 1: With Exactly One Player Present

High Survivor is primarily intended as a game where two or more players compete at the same time. However, as a way to jump-start the game if a table is empty, there is a provision for a single player to play alone at the table. In this instance, there is no dealer rake from the pot. Instead, the dealer participates just like a regular player, and can win the pot if the dealer is the sole High Survivor. In addition, to create a built-in house advantage the dealer takes ties. (In the multiplayer game, ties mean that no player wins the pot, and play continues to the next hand.)

Since the dealer is a normal participant in the game, it is important to insure that a knowledgeable player cannot gain an edge in the game by choosing his target suit based on the remaining deck composition. To avoid this threat, the sole player's suit choice also applies to the dealer, eliminating the benefit such a player could otherwise gain. Therefore, after the player chooses a suit, a card is dealt to the player and a card is dealt to the dealer, and they are both compared to the suit designated by the player to determine which players survive.

The easiest way to calculate the house edge in this case is to examine the complete array of possible outcomes. To keep the calculations simple, we'll treat every individual card of a 6-deck shoe as unique, although in reality the number of distinguishable combinations is much smaller.

If we begin by assuming that a player's card is equally likely to be any of the 312 cards in a 6-deck shoe, and the dealer's card is then equally likely to be any of the 311 remaining cards, we can see that the common denominator of our calculations should be (312 * 311), or 97,032. Now we'll examine how many of each of those possible 97,032 hands results in each outcome.

Outcome:	Number of Ways	
Neither player matches the target suit:	234 * 233	= 54,522
Player matches, dealer does not:	78 * 234	= 18,252
Dealer matches, player does not:	78 * 234	= 18,252
Both dealer and player match, no tie:	78 * 72	= 5,616
Both dealer and player match, and tie:	78 * 5	= 390
	TOTAL:	= 97,032

Using these totals, we can calculate the average size of a won jackpot. In 97,032 hands, 194,064 units will be wagered by both players combined. Only 42,510 of those games results in a win by either party, which implies that the average won jackpot is 194,064/42,510 = 4.565 units. Over the course of all these games, the dealer wins 390 more matches than the player, for a house profit of 390 X 4.565 = 1780.40 units. Traditionally, house edge is stated in terms of the percentage of player wagers made, so that works out to 1780.40 / 97032 = 1.8349%.

The house edge on the single player version of High Survivor is 1.8349%.

Case 2: With Three or More Players Present

In a multi-player situation, the High Survivor rules provide that the dealer collects a rake of a single wager unit from the pot, on each hand where the dealer matches his designated suit. The dealer's suit is designated by a single card dealt from the shoe after all player bets are placed, but before any player has received their card. The probability that the dealer's suit will match the dealer's suit designator card is easily calculated:

[Number of Decks * 13] - 1

[Number of Decks * 52] - 1

For a 6 deck game, this yields 77/311, or 0.247588

However, that calculation alone doesn't tell us how often the dealer will collect a rake because if the pot is empty, no rake can be collected. That can occur when every player matches their selected suit, and no player's ante bet is moved into the jackpot. (Another requirement for the jackpot to now be empty is that the jackpot was empty after the conclusion of the prior hand. Otherwise, the dealer is able to collect a rake despite no player contributing to the jackpot on this hand.)

In estimating the house edge, our primary concern is determining the lowest house edge that can result from player decisions. In this case, the players should each choose different suits to the extent possible, to maximize each other's ability to correctly match their selected suit. (If two players choose the same target suit, the chance that both will succeed is lessened.)

Assuming that players will choose an unused suit when possible, here are the probabilities of no jackpot contribution (all player suits match) in a six-deck game.

Three players: p(3match) = (78/312) X (78/311) X (78/310) = 0.015776372
Four players: p(3match) X (78/309) = 0.003982385
Five players: p(4match) X (77/308) = 0.000995596
Six players: p(5match) X (77/307) = 0.000249710
Seven players: p(6match) X (77/306) = 0.0000628355

The probability that at least one player will contribute to the jackpot in a particular hand is 1.0 less the probabilities just listed. For example, with a 3 players game, the probability of some jackpot contribution this hand is 1.0 - 0.015776372, or 0.98422363.

Figuring the house edge is now a matter of combining the probability of a dealer suit match, and the probability that the jackpot includes something from which to collect the rake. The house rake is constant, no matter how many players are at the table. Therefore, the fewer players involved, the higher the effective house edge. In each case, we'll use the following formula:

$$\frac{p(DealerMatch) \times p(JackpotNotEmpty)}{NumberOfPlayers}$$

Here are the resulting house edges for each number of players at the table:

Three Players:	8.12%
Four Players:	6.17%
Five Players:	4.95%
Six Players:	4.13%
Seven Players:	3.54%

These calculations slightly understate the house edge, because they assume that anytime all players match their suit, no rake is collected. However, if the jackpot was non-empty at the time, a rake can still be collected. This simplification of the calculations has only a minor effect on the house edge.

While it is useful to know the house edge as calculated above, a more relevant statistic is the actual profitability that can be expected from the game. That number is barely impacted by the number of players at the table, and ranges from 0.244 to 0.248 of a bet per hand. For example, at a $5 table, the expected house profit ranges from $1.22 per hand to $1.24 per hand played depending on the number of players at the table.

Case 3: With Exactly Two Players Present

When exactly two players are present in the game, the rules are quite similar to the 3+ player game just described. However, there is one additional exception that comes into play. Consider what happens in this two player game: One of the players matches, the dealer matches with a lower card, and the other player does not match his suit. In that case, the normal rules would have the dealer collect a rake from the pot, but in this case, only one player has contributed to the pot. If the dealer takes the rake, there is no prize to be won by the winning High Survivor. Therefore, in the two-player game only, the dealer will not collect a rake when only a single betting unit is in the jackpot, unless the dealer is the High Survivor. The most common way for that to occur is if one and only one player matches his suit, and the dealer matches with a lower rank.

For the dealer to collect a rake, as always, the dealer's suit must match the dealer's suit designator card. (p=77/311=0.247588)

Then, there are two possibilities:
1) If the dealer is the High Survivor, he will always collect a rake, except when the Jackpot is empty (all players matched, and no jackpot carried over).
2) If the dealer is not the High Survivor, he will collect a rake only if the Jackpot contains two or more bets (with the remaining chips going to the winning player.) The only way a dealer can collect a rake in this case is if the jackpot were carried forward from a previous hand. Otherwise, there's no way for two chips to be in the pot and the dealer not be High Survivor.

We'll designate all individual hand outcomes using these symbols: n = no match, m = match but not high survivor, H = High Survivor. Each hand outcome will be shown in the sequence with the two players first, followed by the dealer. For example, nmH means that the first player did not match their suit, the second player did match, but the dealer matched and was High Survivor.

The possible outcomes are:

Hand	Action	Pot Effect
nnn	Both player bets go in pot.	+2
nnH	Both player bets go in pot, Dlr rakes	+1
nHn,Hnn	1 bet goes in jackpot, other player wins.	Empty
nmH,mnH	1 bet in, dlr rakes.	Unchanged
nHm,Hnm	1 bet in, dlr rakes if 2+ chips in pot, pot won.	Empty
mmH	No bets in, dlr rakes if pot not empty.	-1 (or empty)
mHm,Hmm	No bets in, player wins pot if any.	Empty

If we look through those possible hands, we see that the dealer always rakes for hands nnH, nmH, and mnH. Then, the dealer also rakes for nHm and Hnm, only if 2 or more chips are in the pot (which means that at least one chip must have been carried forward.) Also, the dealer rakes on hand mmH whenever any chip was carried forward.

Calculation of some of these probabilities is straightforward, but the determination of the probability of a jackpot being carried forward is more problematic, and is best approached by simulation methods.

*** Analysis is incomplete ***

Sincerely,
Kenneth R Smith